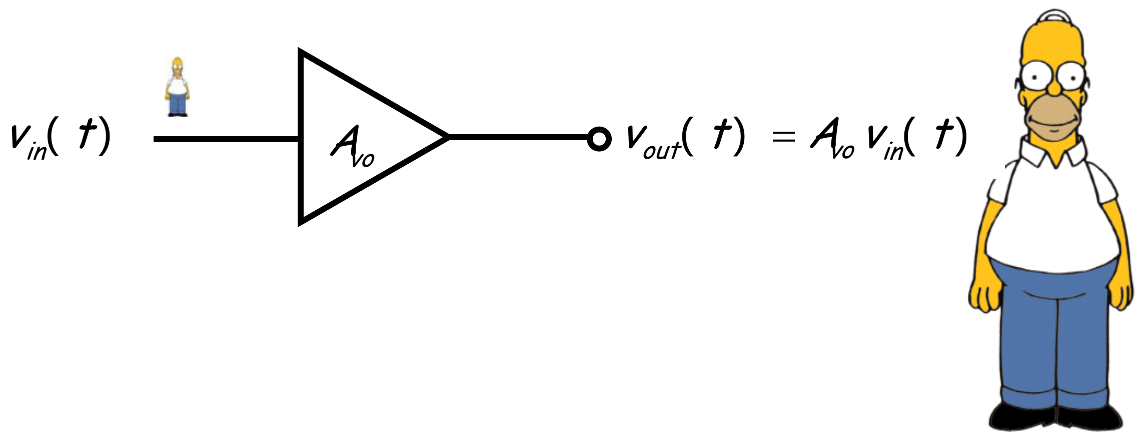
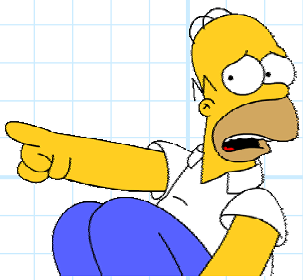
Amplifiers

An **ideal** amplifier takes an input signal and reproduces it **exactly** at its output, only with a **larger** magnitude!



where A_{vo} is the open-circuit **voltage gain** of the amplifier.

Now, let's express this result using our knowledge of **linear circuit theory** !



Recall, the output $v_{out}(t)$ of a linear device can be determined by **convolving** its input $v_{in}(t)$ with the device **impulse response** $g(t)$:

$$v_{out}(t) = \int_{-\infty}^{t} g(t - t')v_{in}(t')dt'$$

The impulse response for the **ideal** amplifier would therefore be:

$$g(t) = A_{vo}\,\delta(t)$$

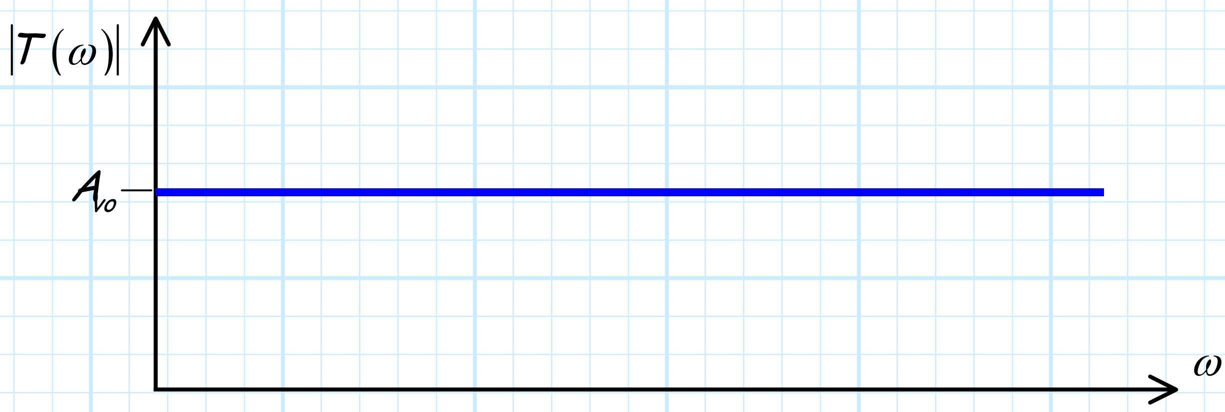
so that:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')v_{in}(t')dt' \\ &= \int_{-\infty}^{t} A_{vo}\delta(t-t')v_{in}(t')dt' \\ &= A_{vo}\,v_{in}(t) \end{aligned}$$

We can alternatively represent the ideal amplifier response in the **frequency domain**, by taking the **Fourier Transform** of the impulse response:

$$\begin{aligned} T(\omega) &= \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt \\ &= \int_{-\infty}^{\infty} A_{vo}\delta(t)e^{-j\omega t}dt \\ &= A_{vo} + j0 \end{aligned}$$

This result, although simple, has an interesting interpretation. It means that the amplifier exhibits gain of A_{vo} for sinusoidal signals of **any** and **all** frequencies!



Moreover, the ideal amplifier does not alter the **relative phase** of the sinusoidal signal (i.e., no phase shift).

In other words, if:

$$v_{in}(t) = \cos(\omega t)$$

then at the output of the ideal amplifier we shall see:

$$\begin{aligned} v_{out}(t) &= \left|T(\omega)\right| \cos\left(\omega t + \angle T(\omega)\right) \\ &= A_{vo} \cos(\omega t) \end{aligned}$$

BUT, there is one **big** problem with an ideal amplifier:

→ They are **impossible** to build !!

Q: *Why is that ??*

A: Two reasons:

a) An ideal amplifier has **infinite** bandwidth.

b) An ideal amplifier has **zero** delay.

Not gonna happen !

Let's look at this **second** problem first. The ideal amplifier impulse response $g(t) = A_{vo}\,\delta(t)$ means that the signal at the output occurs **instantaneously** with the signal at the input.

This of course **cannot** happen, as it takes some small, but non-zero amount of **time** for the signal to propagate through the amplifier. A more **realizable** amplifier impulse response is:

$$g(t) = A_{vo}\,\delta(t-\tau)$$

resulting in an amplifier output of:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{t} g(t-t')\,v_{in}(t')\,dt' \\ &= \int_{-\infty}^{t} A_{vo}\,\delta(t-\tau-t')\,v_{in}(t')\,dt' \\ &= A_{vo}\,v_{in}(t-\tau) \end{aligned}$$

In other words, the output is both an amplified and **delayed** version of the input.

* Ideally, this delay does not **distort** the signal, as the output will have the same form as the input.

* Moreover, the delay for electronic devices such as amplifiers is **very small** in comparison to human time scales (i.e., $\tau \ll 1$ second).

***** Therefore, propagation delay τ is generally **not** considered a **problem** for most amplifier applications.

Let's examine what this delay means in the **frequency domain**.

Evaluating the Fourier Transform of this **modified** impulse response gives:

$$\begin{aligned} T(\omega) &= \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt \\ &= \int_{-\infty}^{\infty} A_{vo}\, \delta(t-\tau)\, e^{-j\omega t}\, dt \\ &= A_{vo} \cos(\omega\tau) + j\, A_{vo} \sin(\omega\tau) \\ &= A_{vo}\, e^{j\omega\tau} \end{aligned}$$

We see that, as with the ideal amplifier, the magnitude $|T(\omega)| = A_{vo}$. However, the relative **phase** is now a linear function of frequency:

$$\angle T(\omega) = \omega\tau$$

As a result, if $v_{in}(t) = \cos(\omega t)$, the output signal will be:

$$\begin{aligned} v_{out}(t) &= |T(\omega)| \cos(\omega t - \angle T(\omega)) \\ &= A_{v} \cos(\omega t - \omega\tau) \end{aligned}$$

In other words, the output signal of a **real** amplifier is **phase shifted** with respect to the input.

In general, the amplifier phase shift $\angle T(\omega)$ will not be a perfectly linear function (i.e., $\angle T(\omega) \neq \omega\tau$), but instead will be a more general function of frequency ω.

However, if the phase function $\angle T(\omega)$ becomes too "non-linear", we find that signal **dispersion** can result—the output signal can be **distorted**!

Now, let's examine the **first problem** with the ideal amplifier. This problem is best discussed in the **frequency** domain.

We discovered that the **ideal** amplifier has a frequency response of $|T(\omega)| = A_{vo}$. Note this means that the amplifier gain is A_{vo} for **all** frequencies $0 < \omega < \infty$ (D.C. to daylight !).

The bandwidth of the ideal amplifier is therefore **infinite** !

* Since every electronic device will exhibit **some** amount of inductance, capacitance, and resistance, every device will have a **finite** bandwidth.

* In other words, there will be frequencies ω where the device does **not work** !

* From the standpoint of an amplifier, "not working" means $|T(\omega)| \ll A_{vo}$ (i.e., **low gain**).

* Amplifiers will therefore have **finite** bandwidths.

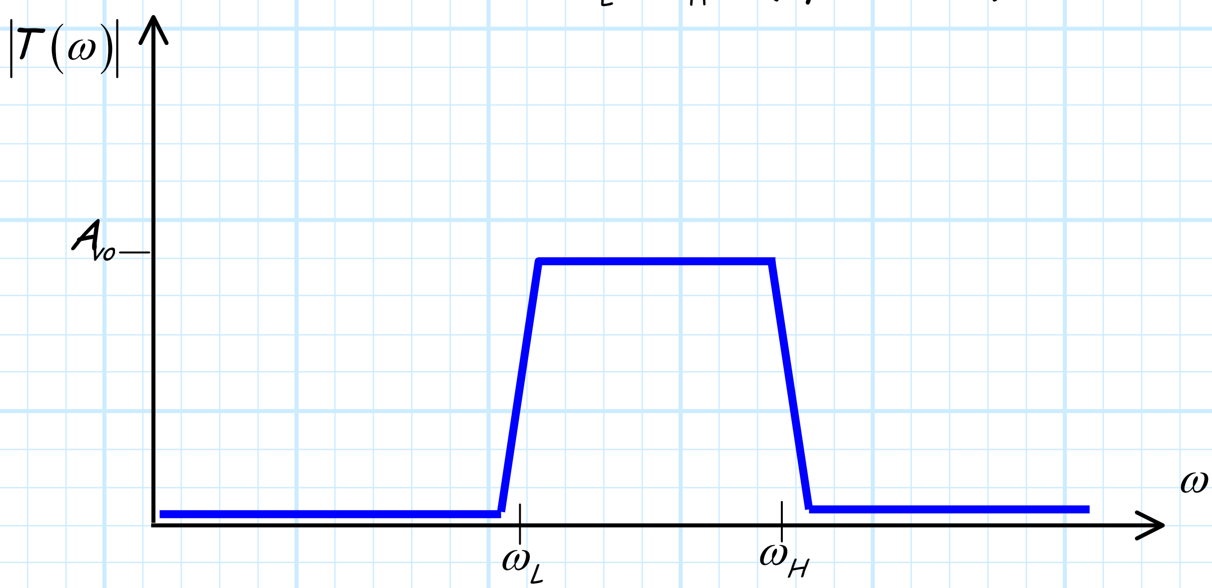
There is a range of frequencies ω between ω_L and ω_H where the gain will (approximately) be A_{vo}. For frequencies outside this range, the gain will typically be small (i.e. $|T(\omega)| \ll A_{vo}$):

$$|T(w)| = \begin{cases} \approx A_{vo} & \omega_L < \omega < \omega_H \\ \ll A_{vo} & \omega < \omega_L, \omega > \omega_H \end{cases}$$

The **width** of this frequency range is called the amplifier **bandwidth**:

$$Bandwidth \doteq \omega_H - \omega_L \quad \text{(radians/sec)}$$

$$\doteq f_L - f_H \quad \text{(cycles/sec)}$$



One result of having a **finite bandwidth** is that the amplifier impulse response is **not** an impulse function !

$$g(t) = \int_{-\infty}^{\infty} T(\omega)\, e^{+j\omega t} dt \neq A_{vo}\, \delta(t-\tau)$$

The **ideal** amplifier is not possible!